

Space-based Orbit Determination for a Lunar Low-Energy Transfer Orbit

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Abstract: The Cislunar DRO Exploration (CDROE) mission is supported by the Strategic Priority Research Program of the Chinese Academy of Sciences, which will launch two satellites (combined) into a Distant Retrograde Orbit (DRO) in cislunar space via a low-energy lunar transfer. It utilizes the gravitational assistance of the Moon and the Sun, and its apoapsis is around 1 million km. The transfer period of the combined satellites is about 4 months and there are tens of trajectory correction maneuvers (TCMs) and insertion correction maneuvers (ICMs), so it is necessary to improve orbit determination accuracy and convergency performance. In addition to traditional ground-based tracking, we propose a novel Space-based orbit determination (OD) strategy that utilizes an inter-satellites radiometric link between an LEO satellite and the combined satellites to determine the transfer orbit. This mission will launch an LEO satellite beforehand equipped with a GNSS receiver, so it could obtain high-accuracy orbit and timing information easily. The advantages of Space-based OD compared with ground-based tracking include rapidly-changing measurement geometry, relatively short invisible tracking gap, and inter-satellite link that are not affected by atmospheric delays. We establish inter-satellite and satellite-ground links using K-band radiometric and dual one-way pseudorange measurement method. The Extended Kalman Filter algorithm using measurement data is employed, which is convenient for onboard computation. Taking the lunar transfer orbit as an example, this paper demonstrates the orbit determination and time synchronization performance using a Space-based OD strategy. High-fidelity simulations using realistic link specifications show that the final statistic position accuracy is about 130 meters and the time synchronization accuracy is in nanoseconds, and the convergence time after the maneuvers is less than 2 hours. The result shows that the Space-based OD strategy is a promising technique for cislunar spacecraft navigation.

Keywords: Cislunar Space; Space-based Orbit Determination; LEO; Dual one-way ranging; Inter-satellite link.

1. Introduction

Cislunar Distant Retrograde Orbit (DRO) is a type of three-body dynamics orbit that simultaneously rotates the Earth and the Moon, it is a critical orbit resource of the cislunar high

gravitational potential area and a hub that connects the Earth, the Moon, and deep space [1]. DRO possesses a series of unique spaceflight dynamics nature, a spacecraft in the orbit can steadily stay for 10 years without maneuvering, DRO is the best starting place for spacecraft to reach any other place in cislunar space. The Strategic Priority Research Program of the Chinese Academy of Sciences has been supporting the Cislunar DRO Exploration (CDROE) mission, which plans to send 3 small satellites to different cislunar orbits to establish a large experimental constellation. During the process, validation of low-energy transfer and long-term stability will be accomplished. CDROE will carry out missions including space-based orbit determination mission based on inter-satellite link tracking measurement, cislunar spacecraft autonomous cruising navigation mission, and other missions. This study focus on the orbit determination method of low-energy DRO transfer orbit.

Autonomous and precise orbit determination services are important for these spacecraft to work normally. The traditional orbit determination methods used in practice are mainly relying on GNSS or ground station (Deep Space Network). Current navigation solutions based on GNSS and ground stations are not able to meet the navigation needs of these spacecrafts. The GNSS primarily provides service for spacecraft near the Earth, not specifically designed for cislunar spacecraft. More cost is required for cislunar spacecraft to accomplish orbit determination using GNSS signal [2]. It is difficult for ground stations to support real-time orbit determination of cislunar spacecraft due to poor measurement geometry which is caused by stations' slow movement in the inertial reference frame, that's why the orbit determination solution mainly employs the least square batch processing method [3]. To solve the problem caused by the deficiency of these navigation solutions, the orbit determination method using an inter-satellite link and satellite-ground link is proposed in this study, the mission scenario is illustrated in Fig.1. An LEO satellite that is equipped with a GNSS receiver can easily obtain precise real-time orbit states after it receives the GNSS navigation signals, so it can be used to navigate other spacecraft.

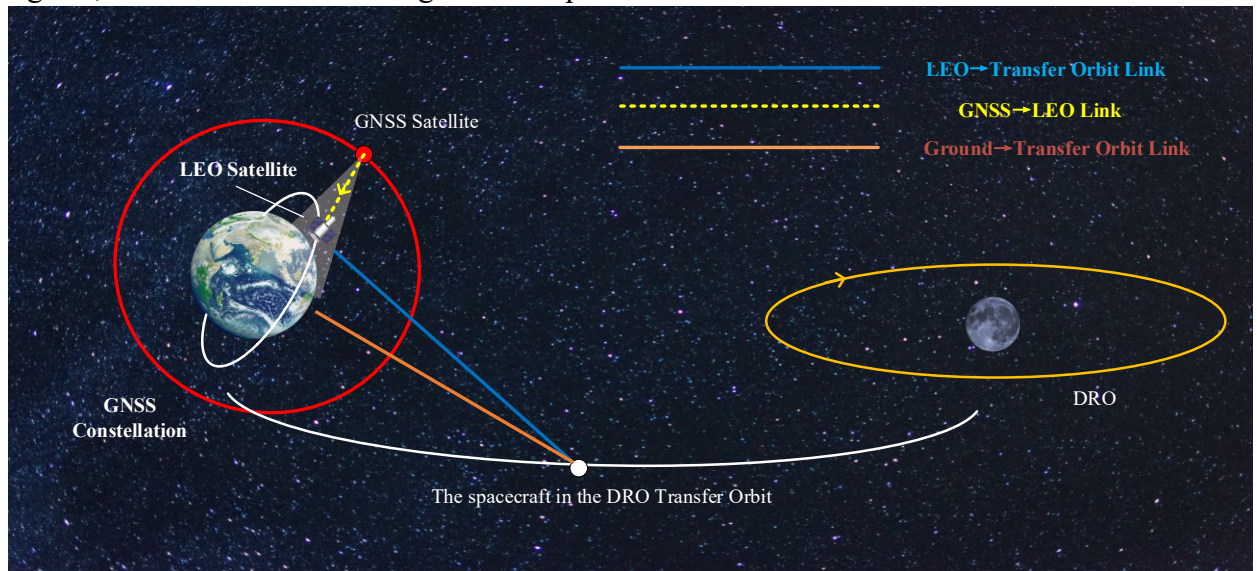


Figure 1. The mission scenario of the space-based orbit determination method

Figure 2 illustrates the comparison of range variation between inter-satellite link and satellite-ground link. Compared with ground stations, LEO satellites possess better measurement

conditions because of their rapid motion characteristic: the link block time caused by the Earth is shorter; LEO satellites can obtain precise real-time states using GNSS; the measurement link is rarely affected by the atmosphere. These advantages facilitate orbit determination of cislunar spacecraft. Therefore, the inter-satellite and satellite-ground links established between the cislunar spacecraft and an LEO satellite are employed in the proposed orbit determination method.

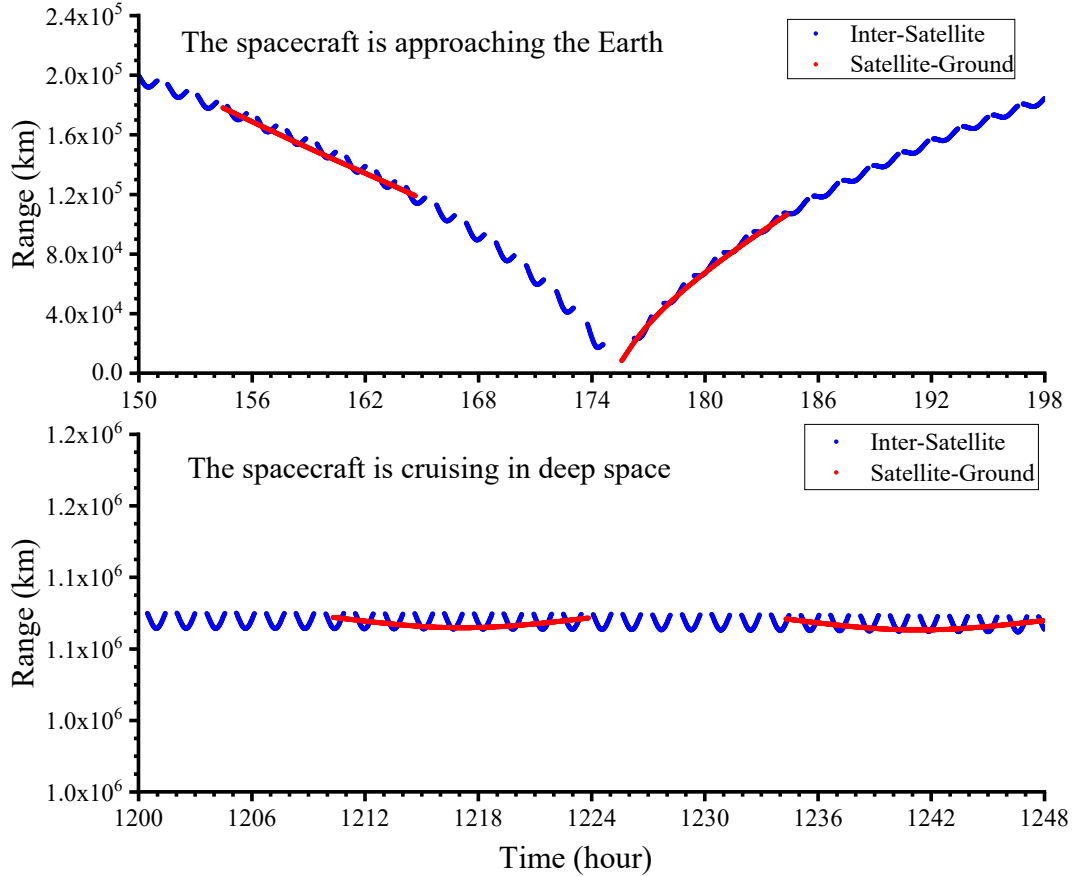


Figure 2. Range variation comparison between inter-satellite link and satellite-ground link
This paper will elaborate all contents in many parts: mission orbit and payload configuration; dual one-way ranging measurement model; orbit determination method; result and analysis; conclusion and discussion.

2. Mission Orbit and Payload Configuration

2.1. Low-Energy Transfer Orbit

CDROE will launch three satellites into space, one of them will be in LEO, other two satellites are combined to transfer along a low-energy trajectory, ultimately staying at DRO. The transfer orbit is illustrated in Fig. 3. The green curve is 2:1 DRO, the red cross symbols represent maneuver locations and the blue curve represents the transfer orbit from near the Earth to DRO. After satellite rocket separation, the combination firstly enters early stage of the transfer orbit. It will orbit the Earth for 2 periods in a high elliptical orbit with two perigee maneuvers. In the next stage, to reduce the energy consumption, considering the gravitational forces of the Sun, the Earth, and the Moon,

the combination of two cislunar satellites will be transferred from satellite rocket separation location to DRO insertion location in approximate 130 days with the furthest distance of more than one million kilometers from the Earth, the orbit is a kind of low-energy transfer orbit [4]. Multiple stages require high accurate orbit determination result in the whole transfer process: near two perigee maneuvers; near the moon; located in deep space.

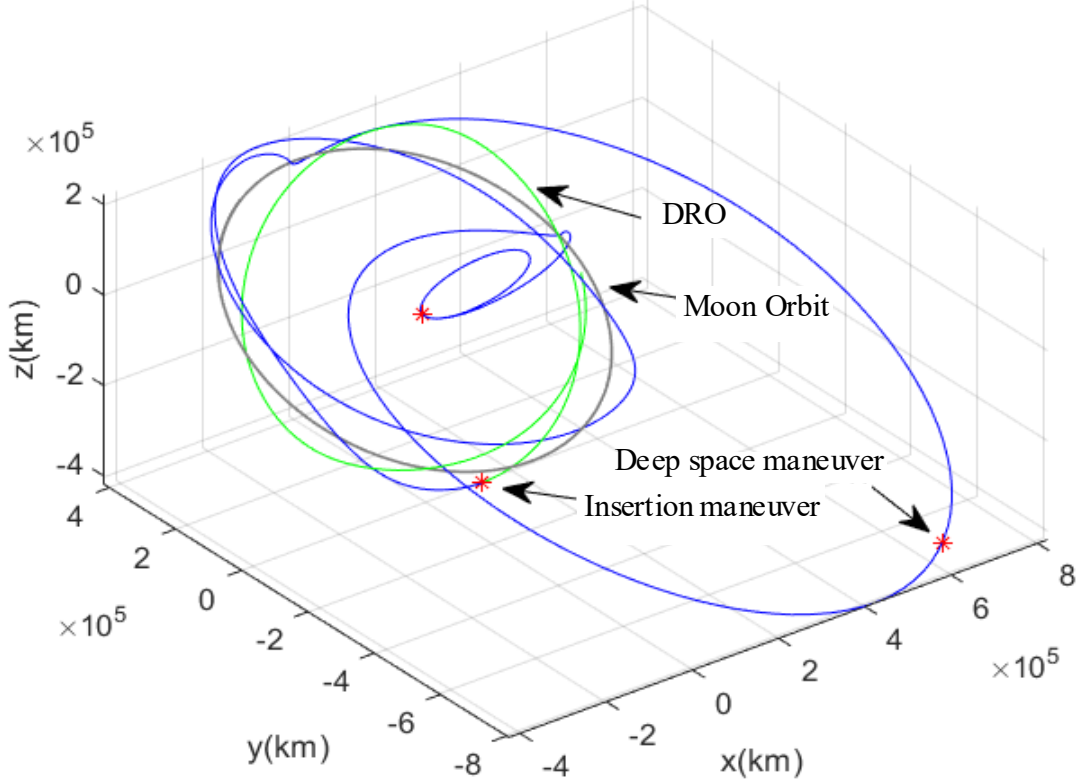


Figure 3. Low-energy DRO transfer orbit (J2000 Reference frame)

Multiple maneuvers including transferring maneuver, orbit correction maneuver, deep space maneuver, and DRO insertion maneuver are conducted to ensure it arrives in the DRO orbit precisely. The satellite combined object will be launched into space at 2023 OCT 04 21:35:24.027344 (UTC), and later enters a highly elliptical orbit, after orbiting for one period, it reaches the first perigee maneuver location at 2023 OCT 12 04:59:00.204002 (UTC), the maneuver magnitude is 10.59m/s, then similarly reaches the second perigee maneuver location at 2023 OCT 18 14:00:09.470894, the maneuver magnitude is 27.11m/s; after some time's traveling, the combined object conducts the deep space maneuver of 9.5m/s magnitude at 2023 NOV 23 19:16:10.862396 (UTC) and finally conducts the insertion maneuver of 59.63m/s magnitude at 2024 FEB 11 13:48:35.434387 and later enters in DRO. Total cruising time is 129.7 days; total maneuver is 138.98m/s. Tab. 1 shows the major maneuvers of the transfer orbit.

Table 1. Major maneuvers of the low-energy transfer orbit

Event	Epoch (UTC)	Velocity impulse (m/s)
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Satellite rocket separation maneuver	T0: 2023 OCT 04 21:35:24.027344	--
TCM	T0+ 1.95713 days	0.81
TCM	T0+ 3.64859 days	0.32
TCM	T0+ 5.32256 days	1.81
Perigee maneuver (1st)	T0+ 7.30806 days	10.59
TCM	T0+ 7.94477 days	7.67
TCM	T0+10.498191 days	10.19
TCM	T0+ 13.067723 days	8.63
Perigee maneuver (2nd)	T0+ 13.683859 days	27.11
TCM	T0+ 17.992863 days	2.58
Deep space maneuver	T0+ 49.903320 days	9.50
Insertion maneuver	T0+ 129.675826 days	59.63

2.2 Payload Configuration

Three small satellites and ground stations adopt a K-band microwave receiving and transmitting system to realize link construction and measurement tracking. The signal transmission payload configuration of the satellites and the ground station is shown in Tab. 2. Some parameters in the table will be used to calculate the measurement model noise of pseudorange data.

Table 2. K-band measurement payload

Platform	Antenna aperture (m)	Frequency transmission (GHz)	Frequency reception (GHz)	EIRP (dBW)	G/T (dBK)
Combination	0.22	26.5	23	40.4	1.61
LEO satellite	0.6	23	26.5	50	11.2
Ground station	2.4	23	26.5	67	24.2

The thermal noise standard deviation of inter-satellite link and satellite-ground link can be calculated as follows [5]:

$$\sigma_{\varepsilon} = \frac{\lambda_K}{2\pi} \sqrt{\frac{B_n}{C/N_0} \left(1 + \frac{1}{2TC/N_0} \right)} \quad (1)$$

where λ_K is the wavelength of K-band microwave; B_n is carrier loop noise bandwidth; T is prediction integration time; C/N_0 carrier to noise power expressed as a ratio, its calculation is expressed as follows [6]:

$$C/N_0 = 10^{\frac{(C/N_0)_{dB}}{10}} \quad (2)$$

$$(C/N_0)_{dB} = EIRP + G/T - LFS + 228.6 \quad (3)$$

where $EIRP$ is effective isotropic radiated power and G/T is the figure of merit, their value can be found in Tab. 3; LFS is free-space path loss, and its calculation is expressed as follow:

$$LFS = 20 \log(f) + 20 \log(r) + 20 \log\left(\frac{40\pi}{3}\right) \quad (4)$$

where f is the frequency in GHz; r is the range between sender and receiver in meter.

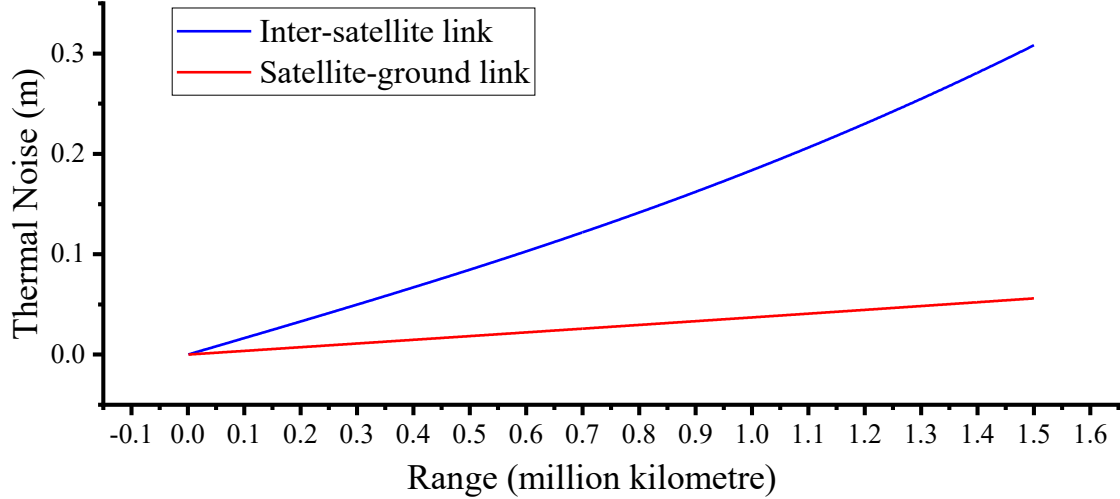


Figure 4. Pseudorange measurement noise of inter-satellite and satellite-ground link

The range from satellite combined object to the Earth varies from a few thousand kilometers to a million kilometers, so the thermal noise varies with the location. The thermal noise variation of the pseudorange of the inter-satellite link and the satellite-ground link is illustrated in Fig. 4.

Another correlating payload of satellite combination is the rubidium clocks, they ensure high-quality operation of communication and measurement by providing accurate time systems. The specification of a small rubidium clock is described in Tab. 3.

Table 3. Major specification of rubidium clock

Number	Specification item		Value
1	Output frequency		10MHz
2	Frequency accuracy (A1)		$\leq \pm 5 \times 10^{-9}$
3	Frequency drifting rate (A2)		$\leq 5 \times 10^{-12}/d$
4	Frequency stability	1s	$\leq 3 \times 10^{-11}$
		10s	$\leq 1 \times 10^{-11}$
		100s	$\leq 3 \times 10^{-12}$
		1000s	$\leq 1 \times 10^{-12}$
		10000s	$\leq 3 \times 10^{-13}$

3. Dual One-Way Ranging Measurement Model

In Dual One-Way Ranging (DOWR) system, satellites in the satellite link send and receive signals to and from each other at different frequency points. Using DOWR measurement information can decouple clock information from orbit position information, therefore the interstellar clock can synchronize time without position residuals converging. Navigation observability can be greatly improved. In the satellite link, if a satellite has poor clock accuracy and stability, it can be time synchronized by another satellite that has a high accuracy and high-stability clock. However, it can only get time synchronized in this system, two satellites' clocks will drift synchronously

without an external clock source. The LEO satellite can receive GNSS navigation signal, and GNSS reference clock resource can correct clocks in the satellite link.

3.1. Clock Model

The clock model includes deterministic and stochastic errors. It is assumed that the clock onboard the DRO satellite is an atomic clock, and deterministic clock errors are usually modeled as a polynomial series form of second order. So, the clock model of the DRO satellite can be expressed as follow:

$$\begin{aligned}\delta\tau_x(t_x) &= A_0^x(t_0) + A_1^x(t_x - t_0) + 0.5 \times A_2^x(t_x - t_0)^2 + \zeta_x(t_x) \\ \dot{\delta\tau}_x(t_x) &= A_1^x + A_2^x(t_x - t_0) \\ \ddot{\delta\tau}_x(t_x) &= A_2^x\end{aligned}\quad (5)$$

Where the t_0 denotes reference time. And $t_x - t_0$ is controlled in 1 day to ensure the accuracy of clock modeling. $\zeta_x(t_x)$ denotes the stochastic clock noise.

In this study, the LEO satellite is equipped with a crystal oscillator with high stability, and the LEO satellite can receive a GNSS navigation signal. Based on the clock discipline system, the timing result can be seen as random, and the timing accuracy is 20 ns(1 σ).

3.2. Ranging Model

After the inter-satellite link and satellite-ground link are established, all of these three platforms can initiate measurement procedures. The measuring signal is bidirectional, thus both the two ends of one link receive the measuring signal to generate pseudorange measurement data. For each measurement process, ignoring the error caused by atmosphere, the sender and receiver are denoted as SD and RE, then the pseudorange can be expressed as follows:

$$R_{RE}^{SD}(t_{RE}) = \|\mathbf{r}_{RE}(t_{RE}) - \mathbf{r}_{SD}(\tilde{t}_{RE})\| + c \cdot (D_{RE}^R + D_{SD}^S) + c \cdot (\delta\tau_{RE}(t_{RE}) - \delta\tau_{SD}(\tilde{t}_{RE})) + \varepsilon_{RE} \quad (6)$$

where t_{RE} and $\tilde{t}_{RE}$ denote receiving and sending epoch; $\mathbf{r}_{RE}$ and $\mathbf{r}_{SD}$ represent the position vector of receiver and sender; c is the speed of light; D_{RE}^R is instrument delay of the receiver at receiving epoch; D_{SD}^S is instrument delay of the sender at sending epoch; $\delta\tau_{RE}$ and $\delta\tau_{SD}$ denote clock offset of receiver and sender; ε_{RE} is thermal noise which is modeled as White Gaussian Noise, its standard deviation can be calculated using specifications of radio wave transmission payload.

In the DOWR measurement model, the two ends of a measurement link are denoted as X and Y. According to Eq. (6), obtained pseudorange data is expressed as follows:

$$R_X^Y(t_X) = \|\mathbf{r}_X(t_X) - \mathbf{r}_Y(\tilde{t}_X)\| + c \cdot (D_X^R + D_Y^S) + c \cdot (\delta\tau_X(t_X) - \delta\tau_Y(\tilde{t}_X)) + \varepsilon_X \quad (7)$$

$$R_Y^X(t_Y) = \|\mathbf{r}_Y(t_Y) - \mathbf{r}_X(\tilde{t}_Y)\| + c \cdot (D_Y^R + D_X^S) + c \cdot (\delta\tau_Y(t_Y) - \delta\tau_X(\tilde{t}_Y)) + \varepsilon_Y \quad (8)$$

3.3. Sum Linear Combination model of DOWR

The sum combination of DOWR pseudorange measurement is summed Eq. (7) and Eq. (8), and then divided by 2, which can be expressed as follows:

$$\begin{aligned}
z_{\text{com}}(t_i) &= \frac{R_X^Y(t_X) + R_Y^X(t_Y)}{2} \\
&= \frac{1}{2} \cdot \left[\begin{aligned} &\rho_X^Y + \rho_Y^X + c \cdot (D_X^R + D_Y^S + D_X^S + D_Y^R) + c \cdot (\delta\tau_X(t_X) - \delta\tau_Y(\tilde{t}_X)) \\ &+ c \cdot (\delta\tau_Y(t_Y) - \delta\tau_X(\tilde{t}_Y)) + \varepsilon_X + \varepsilon_Y \end{aligned} \right] \\
&= \frac{1}{2} \cdot \left[\rho_X^Y + \rho_Y^X + c \cdot D_{\text{com}} + c \cdot ((t_X - \tilde{t}_Y) \dot{\delta\tau}_X) + c \cdot \Delta\delta\tau_Y + \varepsilon_X + \varepsilon_Y \right]
\end{aligned} \tag{9}$$

Where the $D_{\text{com}} = c \cdot (D_X^R + D_Y^S + D_X^S + D_Y^R)$ denotes the influence of equipment delays in the sum combination. The $\Delta\delta\tau_Y$ represent the variation of clock offset in the period of $t_Y - \tilde{t}_X$. The variation of the Rubidium clock's frequency presents the characteristic of white noise, and the stochastic clock noise after integration belongs to the random walk model, which has a time-correlated characteristic, the value of stochastic clock noise is not very big. Because the stochastic clock noise is continuous and almost cancels out with opposite signs in the summation combination, the influence can be eliminated.

It is easy to find that the sum combination can eliminate the fixed difference in clock offset and stochastic errors. The remaining elements include the sum of geometric distances in different directions, the clock drift when a signal transmitting from one satellite to another, and thermal noise in the measurement.

In the study, the Rubidium clock is used in the DRO satellite, so the clock frequency drift A_2^X is about $3\text{E-}13/\text{day}$, the affection can be ignored. When it comes to the Hydrogen maser, the clock frequency drift is smaller, the accuracy is about $5\text{E-}15/\text{day}$. The clock offset from GNSS is continuous, not completely discrete and random, so in a few seconds, the value $\Delta\delta\tau_Y$ is at the nanometer level.

The equipment delay has a relationship with the satellite's temperature change. So, when the temperature changes slowly, in a short period, equipment delay can be modeled as a constant. In the orbit determination process, equipment delays are estimated to reduce their influence.

3.4. Subtraction Linear Combination model of DOWR

Firstly, the instantaneous distance of the measurement model needs to be corrected, because the two satellites have relative motion, and the geometric distances in different directions are not equal and cannot be directly eliminated. The schematic diagram of signal transmission can be shown in Fig.5. In Fig.5, $\rho_R = \|\mathbf{r}_X(t_X) - \mathbf{r}_Y(t_Y)\|$ represent the instantaneous inter-satellite geometric distance of two satellites at the receiving time, the signal transmission geometric distance $\rho_X^Y = \|\mathbf{r}_X(t_X) - \mathbf{r}_Y(\tilde{t}_X)\|$ and $\rho_Y^X = \|\mathbf{r}_Y(t_Y) - \mathbf{r}_X(\tilde{t}_Y)\|$ need to be corrected to it. $\mathbf{A}_X = \mathbf{r}_X(t_X) - \mathbf{r}_X(\tilde{t}_Y)$ and $\mathbf{A}_Y = \mathbf{r}_Y(t_Y) - \mathbf{r}_Y(\tilde{t}_X)$ are displacement vectors, represent the displacement variation of two satellites during signal transmission time.

The correction of the signal transmission geometric distance ρ_X^Y and ρ_Y^X the instantaneous distance ρ_R is performed as follows:

$$\begin{aligned}
\rho_1^2(t_i) &\approx \rho(t_i) + \mathbf{A}_2 \mathbf{e}_{\text{LOS}} \\
\rho_2^1(t_i) &\approx \rho(t_i) - \mathbf{A}_1 \mathbf{e}_{\text{LOS}}
\end{aligned} \tag{10}$$

Where $\mathbf{e}_{\text{LOS}} = (\mathbf{r}_1(t_i) - \mathbf{r}_2(t_i)) / \rho(t_i)$ denotes the unit vector of the position of LEO satellite pointing to DRO satellite at the signal receiving time. Eq. (10) is to project the displacement vectors to the inter-satellite unit vector. The correction to instantaneous distance will bring errors to the model, which can be shown in Fig. 6. In Fig. 6 ε denotes the correction error. When the distance between two satellites is far, and the velocity vector is nearly perpendicular to the inter-satellite unit vector, the errors will be large. The range of 400000 km causes a transmission time of about 1.5s. In the worst circumstance, the LEO satellite's motion direction is perpendicular to the inter-satellite unit vector, the maximum displacement is 10km, and the largest correction error is about 15cm. But in the most normal situation, the correction error is at the cm level.

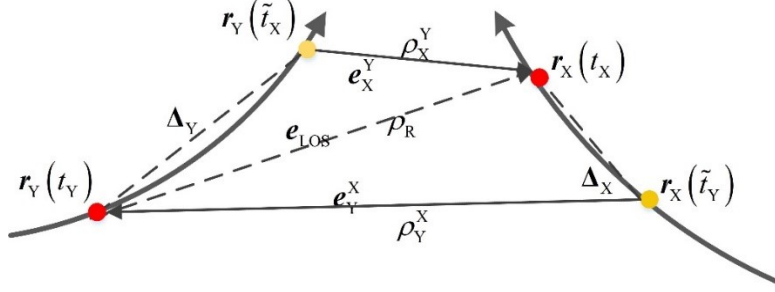


Figure 5. Schematic diagram of signal transmission in DOWR measurement model

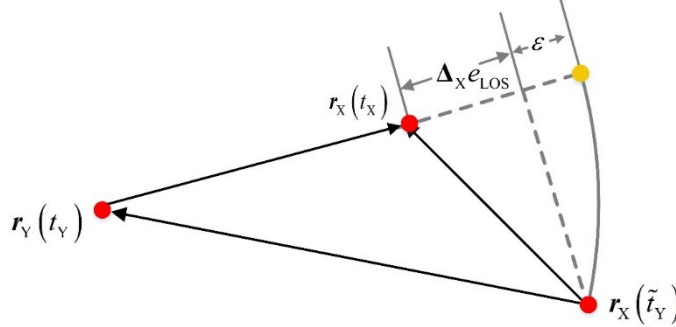


Figure 6. Schematic diagram of Instantaneous range correction in the DOWR system

The subtraction combination of DOWR pseudorange measurement is subtracting Eq. (7) and Eq. (8), and then dividing by 2, which can be expressed as follows:

$$\begin{aligned} z_{\text{sub}} &= \frac{R_X^Y(t_X) - R_Y^X(t_Y)}{2} \\ &= \frac{1}{2} \left[\rho_X^Y - \rho_Y^X + c \cdot (D_X^R + D_Y^S - D_X^S - D_Y^R) \right. \\ &\quad \left. + c \cdot (\delta\tau_X(t_X) - \delta\tau_Y(\tilde{t}_X)) - c \cdot (\delta\tau_Y(t_Y) - \delta\tau_X(\tilde{t}_Y)) + \varepsilon_X - \varepsilon_Y \right] \end{aligned} \quad (11)$$

The $D_{\text{sub}} \doteq D_X^R + D_Y^S - D_X^S - D_Y^R$ denotes the equipment delay term in the subtraction combination, and then brings Eq. (10) to Eq. (11):

$$z_{\text{sub}} \approx \frac{1}{2} \left[(\mathbf{A}_X + \mathbf{A}_Y) \cdot \mathbf{e}_{\text{LOS}} + c \cdot D_{\text{sub}} + c \cdot (\delta\tau_X(t_X) - \delta\tau_Y(\tilde{t}_X)) \right. \\ \left. - c \cdot (\delta\tau_Y(t_Y) - \delta\tau_X(\tilde{t}_Y)) + \varepsilon_X - \varepsilon_Y \right] \quad (12)$$

The clock onboard signal sending time is correct to the signal receiving time by using Taylor expansion to the Quadratic nonlinear item:

$$\delta\tau_X(\tilde{t}_Y) = \delta\tau_X(t_X) - (t_X - \tilde{t}_Y) \dot{\delta\tau}_X(t_X) + 0.5 \times (t_X - \tilde{t}_Y)^2 \ddot{\delta\tau}_X(t_X) \quad (13)$$

Bring Eq. (5) and Eq. (12) to Eq. (13):

$$\begin{aligned}
z_{\text{sub}} &\approx \frac{1}{2} \left[(\mathbf{A}_X + \mathbf{A}_Y) \cdot \mathbf{e}_{\text{LOS}} + c \cdot D_{\text{sub}} + \varepsilon_X - \varepsilon_Y - 2c \cdot \delta\tau_Y \right. \\
&\quad \left. + c \cdot \left(2\delta\tau_X(t_X) - (t_X - \tilde{t}_Y) \dot{\delta\tau}_X(t_X) + 0.5 \times (t_X - \tilde{t}_Y)^2 \ddot{\delta\tau}_X(t_X) \right) \right] \\
&= \frac{1}{2} \left[(\mathbf{A}_X + \mathbf{A}_Y) \cdot \mathbf{e}_{\text{LOS}} + c \cdot D_{\text{sub}} + \varepsilon_X - \varepsilon_Y \right] + c \cdot (\mathbf{A}_0^X - \delta\tau_Y) \\
&\quad + c \left(t_X - t_0 - \frac{\Delta t}{2} \right) \mathbf{A}_1^X + \frac{c}{4} \left((t_X - t_0)^2 + (t_X - t_0 - \Delta t)^2 \right) \mathbf{A}_2^X + \zeta_X(t_X)
\end{aligned} \tag{14}$$

Where Δt denotes the average signal transmission time in the DOWR system, $\Delta t \doteq (\Delta t_X + \Delta t_Y) / 2$. The signal transmission time is nearly equal to each other, so we can have $t_X - \tilde{t}_Y \approx \Delta t$ and $t_Y - \tilde{t}_X \approx \Delta t$. From Eq. (14), we can find that the subtraction combination can eliminate items of inter-satellite ranges, and remain the clock items. The clock offset between the clock onboard the DRO satellite and GNSS timing are $\mathbf{A}_0^X - \delta\tau_Y$, and the equipment delay item is coupled with clock items and not distinguishable, so during the orbit determination process, we will estimate them as a variable parameter. The main errors come from the stochastic clock noise error of clock onboard DRO satellite, GNSS clock offset of LEO satellite, and measurement thermal noise.

3.5. Linearization of Sum Combination

The linearization of geometric distance $\rho_X^Y + \rho_Y^X$:

$$\rho = \frac{1}{2} (\rho_X^Y + \rho_Y^X) = \frac{1}{2} (\|\mathbf{r}_X(t_X) - \mathbf{r}_Y(\tilde{t}_Y)\| + \|\mathbf{r}_Y(t_Y) - \mathbf{r}_X(\tilde{t}_X)\|) \tag{15}$$

Linearization is performed at the known states $\mathbf{r}_{X0}(t_i), \mathbf{v}_{X0}(t_i)$ and $\mathbf{r}_{Y0}(t_i), \mathbf{v}_{Y0}(t_i)$ at the signal receiving time:

$$\rho(t_i) = \frac{1}{2} \left(\rho_0(t_i) + \frac{\partial h(t_i)}{\partial \mathbf{r}_X(t_i)} \Delta \mathbf{r}_X + \frac{\partial h(t_i)}{\partial \mathbf{v}_X(t_i)} \Delta \mathbf{v}_X + \frac{\partial h(t_i)}{\partial \mathbf{r}_Y(t_i)} \Delta \mathbf{r}_Y + \frac{\partial h(t_i)}{\partial \mathbf{v}_Y(t_i)} \Delta \mathbf{v}_Y \right) \tag{16}$$

Where Δ represents the updated value of the estimated parameters, then we have $\Delta \mathbf{r} = \mathbf{r}(t) - \mathbf{r}_0(t)$, $\Delta \mathbf{v} = \mathbf{v}(t) - \mathbf{v}_0(t)$, and $\rho_0(t_i) = \|\mathbf{r}_{X0}(t_X) - \mathbf{r}_{Y0}(\tilde{t}_X)\| + \|\mathbf{r}_{Y0}(t_Y) - \mathbf{r}_{X0}(\tilde{t}_Y)\|$. The Partial derivative is as follows:

$$\begin{aligned}
\frac{\partial \rho(t_i)}{\partial \mathbf{r}_X(t_i)} &= \frac{1}{2} (\mathbf{e}_X^Y(t_i)^T - \mathbf{e}_Y^X(t_i)^T \cdot \boldsymbol{\Phi}_{rr}^X(t_i)) \\
\frac{\partial \rho(t_i)}{\partial \mathbf{v}_X(t_i)} &= -\frac{1}{2} (\mathbf{e}_X^Y(t_i)^T \cdot \boldsymbol{\Phi}_{rv}^X(t_i)) \\
\frac{\partial \rho(t_i)}{\partial \mathbf{r}_Y(t_i)} &= \frac{1}{2} (\mathbf{e}_X^Y(t_i)^T - \mathbf{e}_Y^X(t_i)^T \cdot \boldsymbol{\Phi}_{rr}^Y(t_i)) \\
\frac{\partial \rho(t_i)}{\partial \mathbf{v}_Y(t_i)} &= -\frac{1}{2} (\mathbf{e}_X^Y(t_i)^T \cdot \boldsymbol{\Phi}_{rv}^Y(t_i))
\end{aligned} \tag{17}$$

Where $\mathbf{e}_Y^X(t_i) = (\mathbf{r}_Y(t_i) - \mathbf{r}_X(t_i - \tau_X^Y)) / \rho_Y^X(t_i)$ and $\mathbf{e}_X^Y(t_i) = (\mathbf{r}_X(t_i) - \mathbf{r}_Y(t_i - \tau_X^Y)) / \rho_X^Y(t_i)$ are unit vectors, the state transition matrix is as follows:

$$\begin{aligned}\Phi^X(\tilde{t}_X, t_X) &= \begin{pmatrix} \Phi_{rr}^X & \Phi_{rv}^X \\ \Phi_{vr}^X & \Phi_{vv}^X \end{pmatrix} \\ \Phi^Y(\tilde{t}_Y, t_Y) &= \begin{pmatrix} \Phi_{rr}^Y & \Phi_{rv}^Y \\ \Phi_{vr}^Y & \Phi_{vv}^Y \end{pmatrix}\end{aligned}\quad (18)$$

3.6. Linearization of Subtraction Combination

If the clock parameters of the satellite at one end of the inter-satellite link are known, the clock estimated parameters represent the absolute clock. As the clock onboard the LEO satellite can get absolute time by GNSS, so the estimated time parameters are as follow:

$$\begin{aligned}\delta A_0 &= A_0^X - \delta t_2 + \frac{D_{\text{sub}}}{2} \\ \delta A_1 &= A_1^X \\ \delta A_2 &= A_2^X\end{aligned}\quad (19)$$

And the partial derivatives with respect to clock parameters are as follow:

$$\begin{aligned}\frac{\partial z_{\text{sub}}}{\partial(c\delta A_0)} &= 1 \\ \frac{\partial z_{\text{sub}}}{\partial(c\delta A_1)} &= \frac{t_X + t_Y}{2} - t_0 - \frac{\Delta t}{2} \\ \frac{\partial z_{\text{sub}}}{\partial(c\delta A_2)} &= \frac{1}{4} \left[\left(\frac{t_X + t_Y}{2} - t_0 \right)^2 + \left(\frac{t_X + t_Y}{2} - t_0 - \Delta t \right)^2 \right]\end{aligned}\quad (20)$$

Where c denotes the speed of light, which is generally estimated by multiplying the clock difference parameter with the speed of light.

4. Space-Based Orbit Determination Method

This part introduces orbit dynamics, observability, and Extend Kalman Filter orbit determination algorithm.

4.1. Orbit Dynamics

Orbit dynamics describes the motion of a spacecraft that is freely cruising in space without maneuver. The first-order ordinary differential equation of orbit dynamics is expressed as follows:

$$\begin{cases} \frac{d\mathbf{r}}{dt} = \mathbf{v} \\ \frac{d\mathbf{v}}{dt} = \mathbf{a}_e + \mathbf{a}_m + \mathbf{a}_s + \mathbf{a}_d + \mathbf{a}_{sp} + \mathbf{u} \end{cases}\quad (21)$$

where $\mathbf{r}$ and $\mathbf{v}$ are position and velocity vector of the spacecraft; $\mathbf{a}_e$ 、 $\mathbf{a}_m$ and $\mathbf{a}_s$ are gravitational accelerations of the Earth, the Moon, and the Sun; $\mathbf{a}_d$ is atmospheric drag acceleration; $\mathbf{a}_{sp}$ is solar radiation acceleration; The force model error can be artificially compensated with the acceleration

process noise $\mathbf{u}$ which is modeled as White Gaussian Noise, the value of its standard deviation can be determined by trials to acquire the best orbit determination result. The vectors in Eq. (21) are expressed in the J2000 reference frame. The state vector is defined as $\mathbf{Y} = (\mathbf{r}^T, \mathbf{v}^T)^T$, then Eq. (21) can be simplified as:

$$\frac{d\mathbf{Y}(t)}{dt} = \mathbf{f}(\mathbf{Y}(t), \mathbf{u}(t), t) \quad (22)$$

The transition matrix from epoch t_0 to t can be expressed as:

$$\Phi_Y(t, t_0) = \frac{\partial \mathbf{Y}(t)}{\partial \mathbf{Y}(t_0)} \quad (23)$$

The transition matrix from acceleration process noise to states is Γ_Y , it is expressed as:

$$\Gamma_Y(t, t_0) = \frac{\partial \mathbf{Y}(t)}{\partial \mathbf{u}(t_0)} = \begin{bmatrix} \frac{\Delta t^2}{2} \times \mathbf{I}_{3 \times 3} \\ \Delta t \times \mathbf{I}_{3 \times 3} \end{bmatrix} \quad (24)$$

where Δt is integration step size and $\mathbf{I}_{3 \times 3}$ represents the identity matrix.

4.2. Extend Kalman Filter

Extend Kalman Filter is suitable for real-time orbit determination so it is adopted to realize orbit determination in this study [7]. The estimated states are defined as:

$$\mathbf{X} = (\mathbf{r}^T, \mathbf{v}^T, c\delta A_0, c\delta A_1, c\delta A_2, C_r, cD)^T \quad (25)$$

where $\mathbf{r}$ and $\mathbf{v}$ are position and velocity; δA_0 , δA_1 and δA_2 are relative clock parameters; C_r is solar radiation pressure coefficient; D is total equipment delay. The Extend Kalman Filter has two steps: one is called “time update”, which calculates the state at the next epoch based on the state obtained at the previous epoch; another is named “measurement update”, which corrects the state predicted in “time update” phase with current measurement data. Given the initial state $\mathbf{X}_0$ at epoch t_0 and its corresponding covariance matrix $\mathbf{P}_0$, the “time update” process is expressed as follows:

$$\begin{aligned} \mathbf{X}_{i+1|i} &= \mathbf{X}(t_{i+1}, \mathbf{X}(t_i) = \mathbf{X}_i) + \Gamma_{i+1|i} \mathbf{u}_i \\ \mathbf{P}_{i+1|i} &= \Phi_{i+1|i} \mathbf{P}_i \Phi_{i+1|i}^T + \Gamma_{i+1|i} \mathbf{Q} \Gamma_{i+1|i}^T \end{aligned} \quad (26)$$

where state transition matrix $\Phi_{i+1|i}$ is expressed as follows:

$$\Phi_{i+1|i} = \begin{bmatrix} \Phi_Y(t_{i+1}, t_i) & \mathbf{0}_{6 \times 3} & \frac{\partial \mathbf{r}(t_{i+1})}{\partial C_r(t_i)} & \mathbf{0}_{6 \times 1} \\ \mathbf{0}_{3 \times 6} & \mathbf{I}_{3 \times 3} & \frac{\partial \mathbf{v}(t_{i+1})}{\partial C_r(t_i)} & \mathbf{0}_{3 \times 1} \\ \mathbf{0}_{1 \times 6} & \mathbf{0}_{1 \times 3} & 1 & 0 \\ \mathbf{0}_{1 \times 6} & \mathbf{0}_{1 \times 3} & 0 & 1 \end{bmatrix} \quad (27)$$

Process noise transition matrix $\Gamma_{i+1|i}$ is expressed as follows:

$$\Gamma_{i+1|i} = \begin{bmatrix} \Gamma_Y(t_{i+1}, t_i) \\ \mathbf{0}_{5 \times 3} \end{bmatrix} \quad (28)$$

In Eq. (26), $\mathbf{Q}_c$ is the covariance matrix of process noise $\mathbf{u}_i$, the value of its diagonal elements is set according to the unmodeled force model error. The usage of process noise can prevent bad results caused by too fast convergence of covariance matrix, it also makes the later measurement data continuously affecting the filtering estimation. The actual value of process noise is chosen by multiple trials to obtain the best orbit determination result. The DOWR measurement data at epoch t_{i+1} can be expressed as follow:

$$\mathbf{z}_{i+1} = \begin{bmatrix} \mathbf{z}_{\text{com}}(t_{i+1}) \\ \mathbf{z}_{\text{sub}}(t_{i+1}) \end{bmatrix} \quad (29)$$

The “measurement update” phase use measurement data $\mathbf{z}_{i+1}$ to calculate $\mathbf{X}_{i+1}$ and $\mathbf{P}_{i+1}$ at epoch t_{i+1} , it can be expressed as follow:

$$\begin{aligned} \mathbf{K}_{i+1} &= \mathbf{P}_{i+1|i} \mathbf{H}_{i+1}^T (\mathbf{R}_{i+1} + \mathbf{H}_{i+1} \mathbf{P}_{i+1|i} \mathbf{H}_{i+1}^T)^{-1} \\ \mathbf{X}_{i+1} &= \mathbf{X}_{i+1|i} + \mathbf{K}_{i+1} (\mathbf{z}_{i+1} - \mathbf{h}(\mathbf{X}_{i+1|i})) \\ \mathbf{P}_{i+1} &= (\mathbf{I} - \mathbf{K}_{i+1} \mathbf{H}_{i+1}) \mathbf{P}_{i+1|i} (\mathbf{I} - \mathbf{K}_{i+1} \mathbf{H}_{i+1})^T + \mathbf{K}_{i+1} \mathbf{R}_{i+1} \mathbf{K}_{i+1}^T \end{aligned} \quad (30)$$

where $\mathbf{K}_{i+1}$ is Kalman gain; $\mathbf{R}_{i+1}$ is measurement noise; $\mathbf{h}(\mathbf{X}_{i+1|i})$ is the calculation value of DOWR model using $\mathbf{X}_{i+1|i}$; $\mathbf{P}_{i+1}$ represents the accuracy of estimated state result, it also can be used to analyze the convergence of orbit determination process. The matrix $\mathbf{H}_{i+1}$ is the design matrix of measurement data to the estimated states, it can be expressed as follow:

$$\mathbf{H}_{i+1} = \begin{bmatrix} \frac{\partial \mathbf{z}_{\text{com}}(t_{i+1})}{\partial \mathbf{X}_{i+1|i}} \\ \frac{\partial \mathbf{z}_{\text{sub}}(t_{i+1})}{\partial \mathbf{X}_{i+1|i}} \end{bmatrix} \quad (31)$$

5. Simulation Results and Analysis

5.1. Orbit Parameters

The initial states of the LEO satellite are expressed in Tab. 4. The position and velocity states of the LEO satellite are determined using GNSS. The position and velocity errors are modeled as sine or cosine functions and the clock offset error is modeled as White Gaussian Noise. The error is expressed in Tab. 5. The ground station is located in Miyun, Beijing (40.5° N, 117° E). The force model of low-energy DRO transfer orbit is described in Tab. 6.

Table 4. Initial states of the LEO satellite

Parameter	Value
Semi-major axis / km	6878.137
Eccentricity	0
Inclination / $^{\circ}$	97.4065
The argument of perigee / $^{\circ}$	0
The right ascension of ascending node / $^{\circ}$	10.3886
True anomaly / $^{\circ}$	0
Period/hour	1.5

Table 5. States error of LEO satellite

Parameter	Value
Position error (m, R/T/N)	1/1/1
Velocity error (m/s, R/T/N)	0.001/0.001/0.001
Clock offset error (ns)	20

Table 6. Force model and integrator of low-energy DRO transfer orbit

Force Model and Integrator	Parameters
Gravitational Force of the Earth	Spherical Harmonic function, GGM03S, 21*21
Gravitation Perturbation of the Moon	Spherical Harmonic function, GRGM660, 8*8
Solar Radiation Pressure	Sphere Model, Double Conic Shadow
Perturbation of Mercury, Venus, Mars, and Jupiter	Mass Point
Relativity Effect	Classical Model
Numerical Integrator	RKF78
Integration Step	60s

5.2. Observability

The trajectory of a ground station in the inertial reference frame is similar to that of an LEO satellite that orbits the Earth. Thus, a satellite can be used to support other spacecraft' orbit determination missions when its position and velocity states are known. Once the force environment and initial states are determined, the orbit of the cislunar spacecraft is uniquely obtained. Meanwhile, the measurement data sequence of inter-satellite and satellite-ground links reflects the orbit of the cislunar spacecraft. Different orbit maps different measurement data sequences and vice versa. Adopting the orbit determination algorithm, the orbit states can be calculated using measurement data.

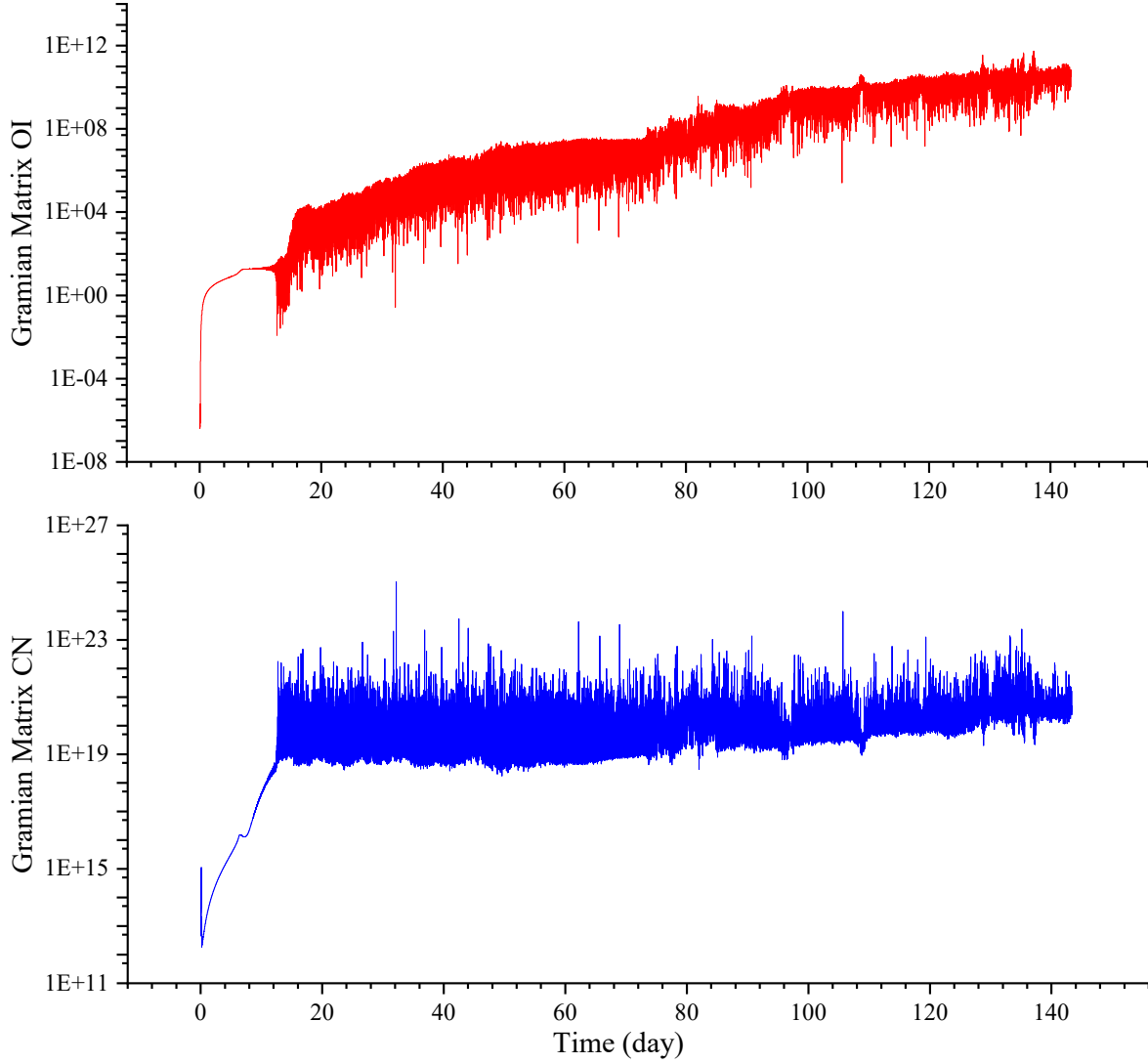


Figure 7. Gramian OI and CN variation of low-energy DRO transfer orbit

Gramian Observability Matrix represents the sensitivity of measurement data to the estimated parameters [8]. For nonlinear discrete-time system, it can be defined as follow:

$$W_d(\mathbf{x}_0, t_0, t_f) = \sum_{k=0}^M \Phi^T(t_k, t_0) \mathbf{H}_k^T \mathbf{H}_k \Phi(t_k, t_0) \quad (32)$$

where M is the number of observations; $\Phi(t_k, t_0)$ is the state transition matrix; $\mathbf{H}_k$ is the design matrix. Generally, the system is observable as long as the gramian matrix is invertible. There are two indicators: Observability Index (OI) and Condition Number (CN). OI is the least eigenvalue of the gramian matrix:

$$\text{OI} = \min \lambda_k(W_d) \quad (33)$$

If its value is too small, the observation error will cause a massive state estimation error. CN is the ratio of maximum eigenvalue to least eigenvalue:

$$CN = \frac{\max \lambda_k(W_d)}{\min \lambda_k(W_d)} \quad (34)$$

If the value of CN is too large, the minor variation of one direction's estimated parameter will submerge the variation of another direction's estimated parameter. That is to say, the system is ill-conditioned if the CN value is too large. OI and CN can be used to assess the observability degree of the system. OI and CN can be used to analyze the precision of estimated parameters and it has nothing to do with the specific method or process. The OI and CN value of low-energy transfer orbit is illustrated in Fig. 7. We can see that influence of the observation error on the estimated parameter is gradually decreasing as the value of OI is gradually increasing; the CN value remains stable after increasing for some time, so the system is not ill-conditioned.

5.3. Measurement Configuration

Considering the actual operating scenario, signal transmission payloads can't work for more than one hundred days, they need to be shut down after some time's work to ensure normal working conditions. During the period when there is no measurement data, the satellite combined object obtains its state through the orbit propagation program, and the position and velocity states errors are propagating in this process. The error propagation is related to the location of the satellite combined object, the initial errors, and the propagation time. Therefore, position error propagation analysis is necessary for the determination of tracking cadence. The calculation equation is as follows:

$$\mathbf{E}(t) = \Phi(t, t_0) \mathbf{E}(t_0) \quad (35)$$

where t_0 and t are propagation start and end epoch; $\Phi(t, t_0)$ is the state transition matrix of position and velocity during the propagation process; the initial state error is expressed as $\mathbf{E}(t_0)$; $\mathbf{E}(t)$ represents state error after propagation. In the analysis, the analysis step size is 6 minutes, initial position and velocity error are set to 100m and 0.01m/s which reflects the normal state error in real conditions.

The analysis result is illustrated in Fig. 8. Too long propagation time will cause too much error in position and velocity, so the tracking process should not be stopped for too long, the propagation time is set to 6 hours. The peaks in the figure are worse propagation conditions, they happen when the combined satellites approach the Earth or the Moon. According to the result, the normal tracking cadence is: stop tracking for 6 hours after 2 hours of continuous measuring. Tracking for 2 hours is sufficient for the satellite combined object to accomplish orbit determination, if not, the tracking time can be extended to 3 or 4 hours. Other continuous tracking cadences are described in the final result illustrated in Tab. 8.

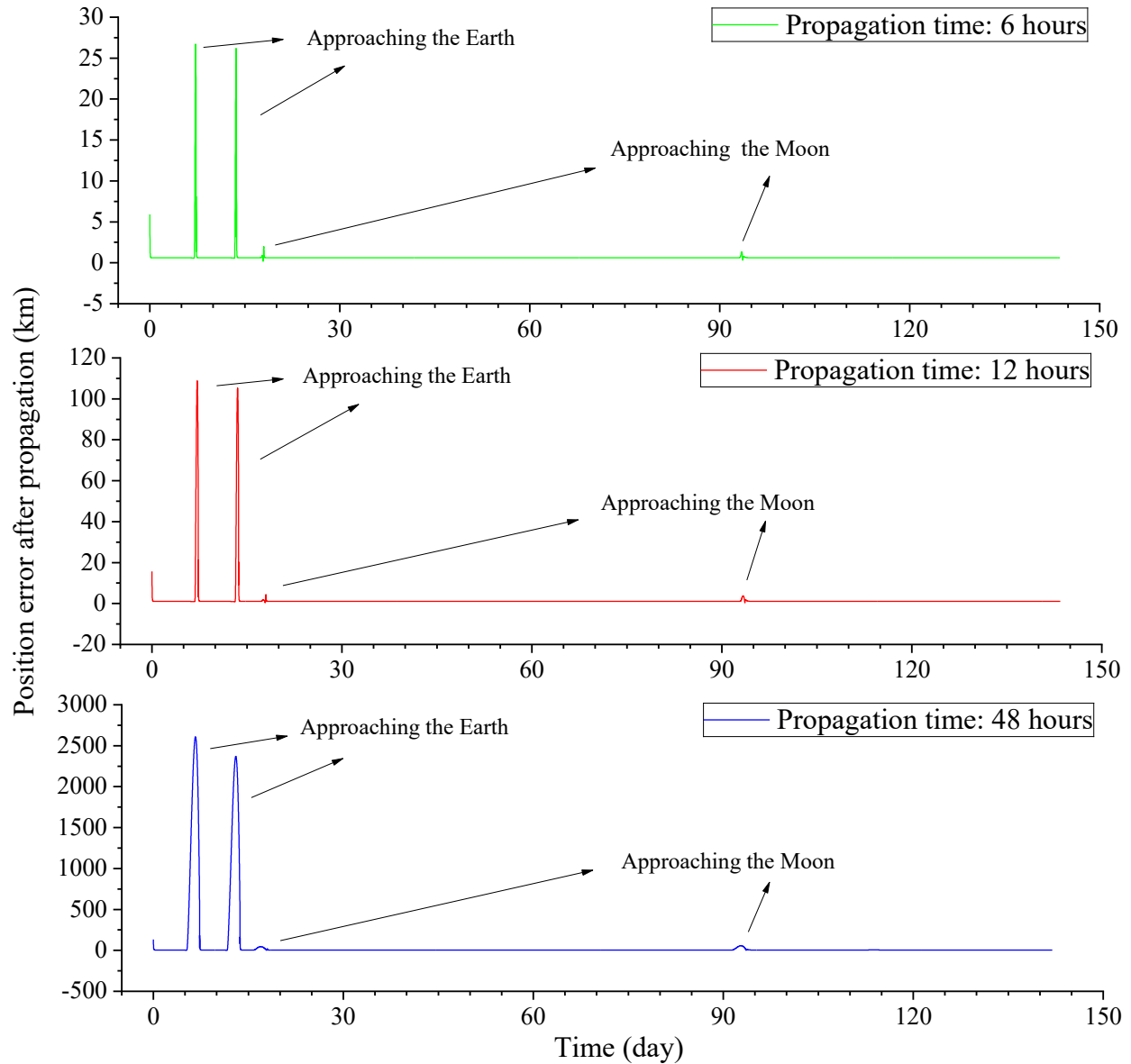


Figure 8. Position error propagation analysis of low-energy DRO transfer orbit (Initial position error: 100m, initial velocity error: 0.01m/s, analysis interval: 6 minutes)

Table 8. Tracking cadence of inter-satellite and satellite-ground links

Mode	Tracking Time
Normal (Discontinuous)	Continuous tracking for 2 hours per 8 hours
Continuous Tracking	Within 12 hours after satellite rocket separation
	From 6 hours before a maneuver to 6 hours after a maneuver
	While approaching the near to the Earth and the near to the Moon

5.4 Navigation Configuration

The force model and integrator used in the orbit determination process are expressed in Tab. 9.

Table 9. Force model and integrator configuration

Force model	Value
Earth Gravity	70×70 GGM01C
Moon Gravity	Mass Point
Sun Gravity	Mass Point
Solar radiation pressure coefficient	1.3
Mass area ratio (m ² /kg)	0.002
Integrator	RungeKutta4
Integration step size (s)	60.0

The initial states and other parameters are expressed in Tab. 10.

Table 10. Configuration of orbit determination

Parameter	Value
Initial position error	50 km (1- σ)
Initial velocity error	5 m/s (1- σ)
Initial clock A0	1e-7
Initial clock A1	1e-10
Initial clock A2	1e-18
Initial Cr and standard deviation	1.3, 0.2 (1- σ)
Initial instrument delay and standard deviation	0 m, 30 m (1- σ)
The standard deviation of acceleration process noise	1e-6 m/s ² (1- σ)
Measurement noise	1 m

The Extend Kalman Filter will be restarted if the maneuver is larger than 10m/s.

5.5. Analysis and Discussion of Simulation Results

The time synchronization result of DRO transfer orbit satellites is illustrated in Fig. 8. It is shown that statistical time accuracy is 3 ns; major clock offset residual is less than 20 ns in the whole orbit determination process. The amplitude of the 20 ns clock offset error is caused by a lack of measurement data. If continuous tracking cadence is employed in the whole orbit determination process, the time synchronization accuracy can be a few nanoseconds or even less than 1 ns. The magnitude of clock offset residual is not varying with the range from satellite combination to the Earth, that's because the DOWR subtraction linear combination decouples range and clock parameters and the evolvement of the clock is independent of range variation.

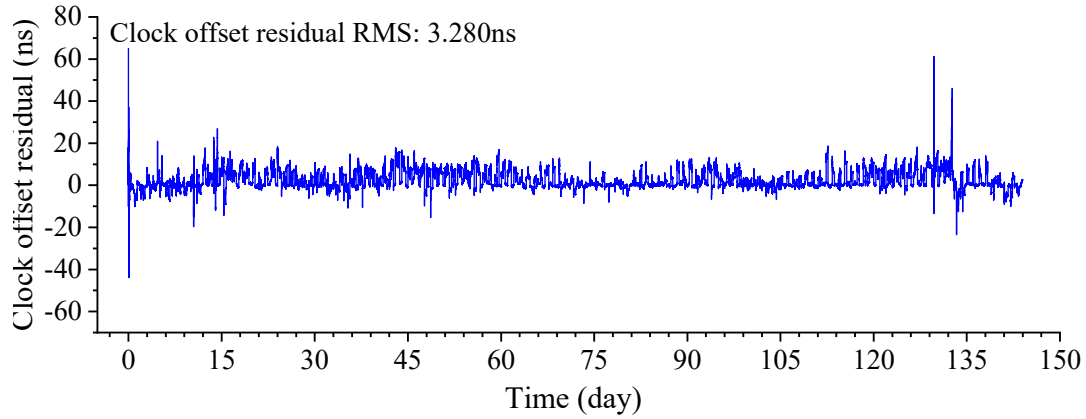


Figure 9. Time synchronization result of low-energy DRO transfer orbit spacecraft

The position determination results are illustrated in Fig. 10. to Fig. 13. The convergence criterion is 1000 meters in these results. Statistic position determination accuracy is 136.13m, but real-time positioning accuracy varies from less than 100 meters near the Earth to more than 300 meters very far from the Earth. The reason for this phenomenon is that the pseudorange error causes more position errors in along-trace and normal directions with the range to the Earth increasing. Or in other words, the distance from the satellite combination to the Earth is further, the less pseudorange error the same position error in along-trace and normal directions gives rise to. The maximum position error caused by maneuver is less than 100 kilometers and the convergence time is less than 2 hours because of the better measurement geometry of the inter-satellite link. After satellite rocket separation and two perigee maneuvers, the position error can easily converge to less than 100 meters in 2 hours.

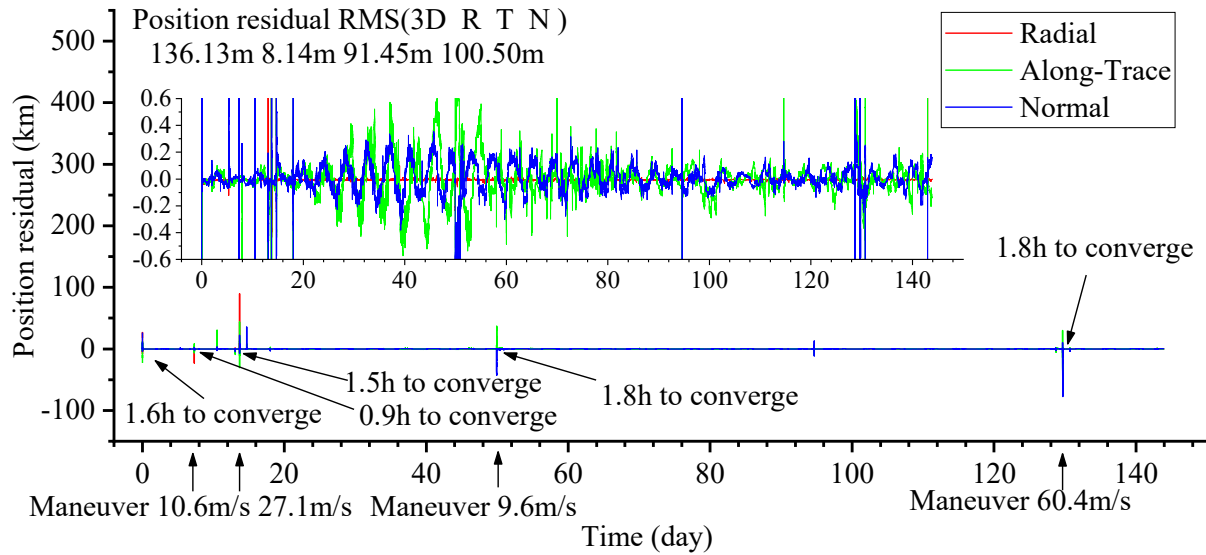


Figure 10. Orbit determination result of low-energy DRO transfer orbit spacecraft

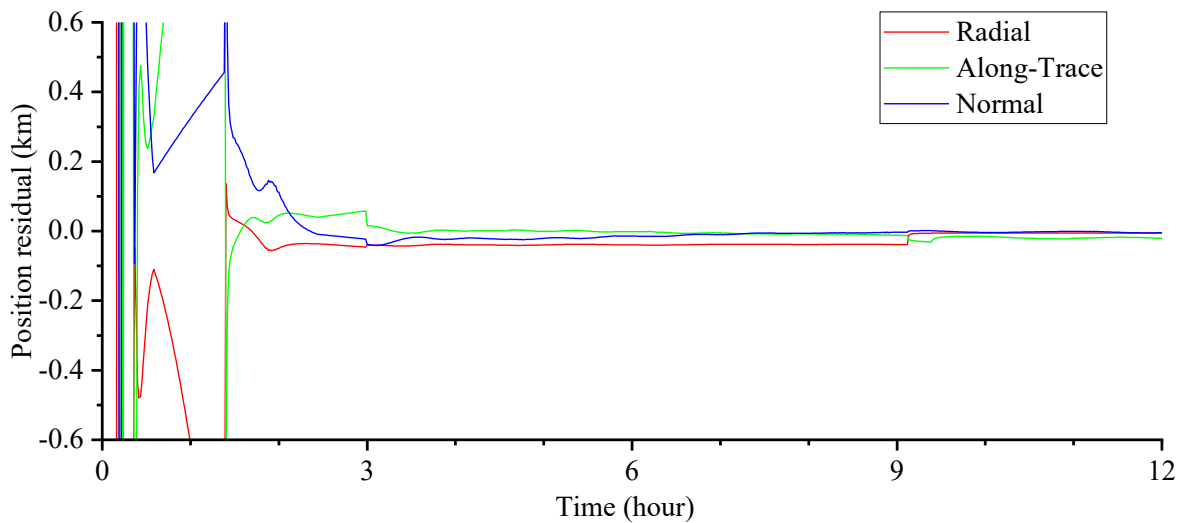


Figure 11. The orbit determination result within 12 hours after satellite rocket separation

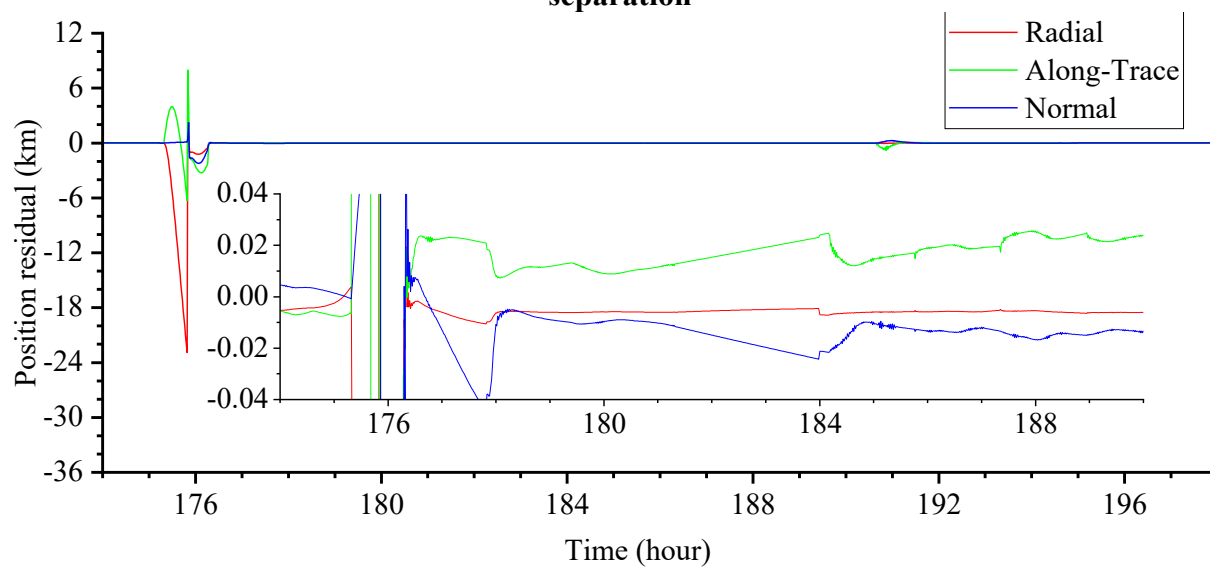


Figure 12. The orbit determination result after the first perigee maneuver

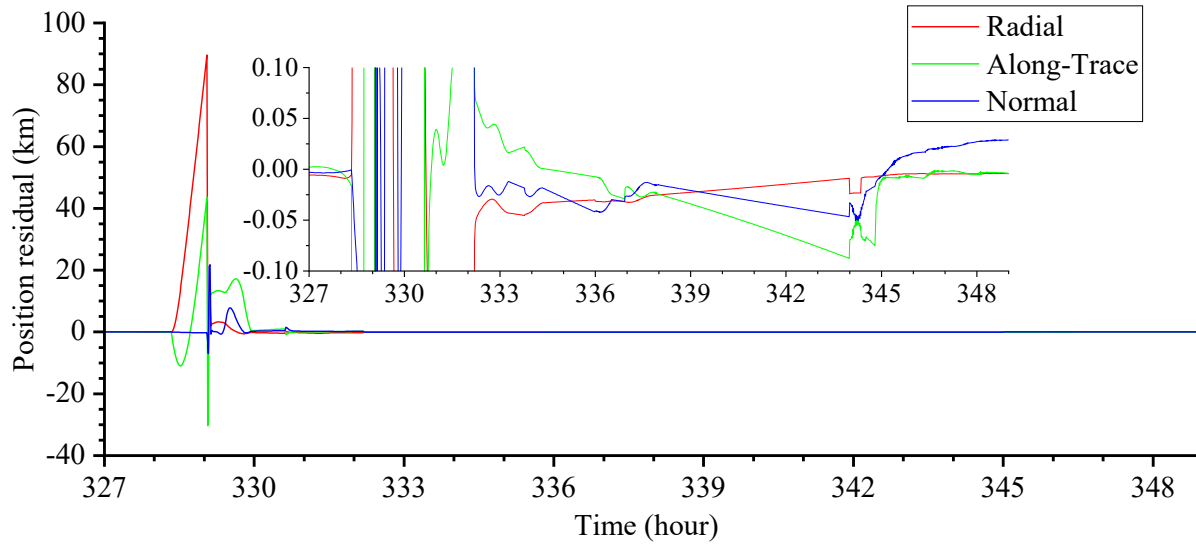


Figure 13. The orbit determination result after the second perigee maneuver

6. Conclusion

This study demonstrates the orbit determination method of low-energy DRO transfer orbit proposed in the CDROE mission. The simulation results show that the final statistic position accuracy is 136.13 meters and the statistic time synchronization accuracy is 3.28 nanoseconds, the convergence time after the maneuver is less than 2 hours. These results prove that this orbit determination solution using the inter-satellite link and the satellite-ground link is feasible and of engineering value. It is worth noting that the satellite-ground link plays a role of supplementary, inter-satellite link plays a major role. The result will be better if more inter-satellite links with LEO satellites are used. It's not difficult to send a satellite to LEO nowadays, so this orbit determination solution is prospective.

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