EKF-SLAM FOR A LIDAR-EQUIPPED QUADCOPTER IN AN ENVIRONMENT OF UNKNOWN LANDMARKS

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Abstract: *In this paper, the problem of Simultaneous Localization and Mapping (SLAM) using Extended Kalman Filter (EKF) is developed for the quadcopter system. We start by presenting the mathematical model of the quadcopter, controlled by the trajectory controller to follow a reference trajectory. Then this paper expounds on the dynamic EKF algorithm assisted by the Light detection and ranging and inertial measurement unit sensors. In the simulation, the quadcopter follows a defined trajectory in the presence of landmarks, sensor noises and field of view, and the results are analysed in terms of Root Mean Square Error (RMSE). As a result, the RMSE for the position of the quadcopter is 10.50 % and the RMSE for the position of landmarks is 3.33 %.*

Keywords: *Quadcopter, Simultaneous Localization and Mapping, Extended Kalman Filter, Linear-Quadratic Regulator.*

1. Introduction

Unmanned Aerial Vehicles (UAVs) have become mainstream nowadays, finding use and importance in not just recreational activities but also in several civilian setups like aerial photography [1], rescue operations [2], logistics [3], etc. Moreover, UAVs are sought to autonomously operate in several unknown environments. The concept of Simultaneous Localization and Mapping (SLAM) can be imparted here such that UAVs can localize themselves based on the measurements it takes of the environment. The sensors can extract feature points and thereby also estimate those features' positions. One of the most employed estimation methods used for non-linear models like that of UAVs is Extended Kalman Filter (EKF), which is the seminal solution to the SLAM problem by Smith and Cheeseman [4]. Thus, this work follows the idea to implement EKF for the estimation of the position of a UAV based on the estimation and mapping of a set of landmarks that are extracted via a three-dimensional (3D) Light Detection and Ranging (LiDAR) sensor. It is assumed that whenever landmarks are initialized, the data association is on-board to uniquely identify all extracted landmarks whenever they are sensed again.

2. System Description and Formulation

2.1. Nomenclature

The variables used in this paper are defined below.

p_n = North Coordinate of the UAV in the inertial frame
p_e = East Coordinate of the UAV in the inertial frame
p_d = Down Coordinate of the UAV in the inertial frame
h = Altitude of the UAV in the inertial frame (Negative of p_d)

$u =$ Velocity component along the x axis (nose) of the UAV in the body frame
$v =$ Velocity component along the y-axis (right wing) of the UAV in the body frame
$w =$ Velocity component along the z-axis (towards earth) of the UAV in the body frame
$\phi =$ Roll angle defined along the x-axis of the UAV in 3-2-1 rotation
$\theta =$ Pitch angle defined along the y-axis of the UAV in 3-2-1 rotation
$\psi =$ Yaw angle defined along the z-axis of the UAV in 3-2-1 rotation
$p =$ Roll rate along the x-axis of the UAV in the body frame
$q =$ Pitch rate along the y-axis of the UAV in the body frame
$r =$ Yaw rate along the z-axis of the UAV in the body frame
$m =$ Mass of the UAV
$J_x =$ Moment of inertia about the x-axis of the UAV in the body frame
$J_y =$ Moment of inertia about the y-axis of the UAV in the body frame
$J_z =$ Moment of inertia about the z-axis of the UAV in the body frame
$F_c =$ Total upward thrust acting on the UAV in the body frame
$\tau_\phi =$ Moment about the roll axis of the UAV in the body frame
$\tau_\theta =$ Moment about the pitch axis of the UAV in the body frame
$\tau_\psi =$ Moment about the yaw axis of the UAV in the body frame
$n =$ Number of landmarks in the environment
$\mathbf{R} =$ Trajectory vector of the UAV = $\begin{bmatrix} p_n & p_e & h & u & v & w & \psi \end{bmatrix}^T$
$\mathbf{L}_k = k^{th}$ landmark's position in the inertial frame
$\mathbf{M} =$ Set of landmark states = $\begin{bmatrix} \mathbf{L}_1^T & \mathbf{L}_2^T & \dots & \mathbf{L}_n^T \end{bmatrix}$
$\mathbf{y}_k =$ LiDAR measurements of the k^{th} landmark in the body frame
$\mathbf{n} =$ Process noise
$\mathbf{v} =$ Measurement noise

2.2. Quadcopter Model

The UAV considered for this project is a quadcopter. A quadcopter includes four rotors that generate propeller forces. Several mathematical models have been proposed to mimic the motion of the quadcopter closely. Among them is the following set of equations of motion [5], which is a six degrees of freedom model for the quadcopter kinematics and dynamics.

$$\begin{bmatrix} \dot{p_n} \\ \dot{p_d} \\ \dot{h} \end{bmatrix} = \begin{bmatrix} \cos\theta\cos\psi & \sin\phi\sin\theta\cos\psi - \cos\phi\sin\psi & \cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi \\ \cos\theta\sin\psi & \sin\phi\sin\theta\sin\psi + \cos\phi\cos\psi & \cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi \\ \sin\theta & -\sin\phi\cos\theta & -\cos\phi\cos\theta \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} \tag{1}$$

$$\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} rv - qw \\ pw - ru \\ qu - pv \end{bmatrix} \begin{bmatrix} -g\sin\theta \\ g\cos\theta\sin\phi \\ g\cos\theta\cos\phi \end{bmatrix} + \frac{1}{m} \begin{bmatrix} 0 \\ 0 \\ -F_c \end{bmatrix} \tag{2}$$

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \frac{\sin\phi}{\cos\theta} & \frac{\cos\phi}{\cos\theta} \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \tag{3}$$

$$\begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} \frac{J_y - J_z}{J_x} qr \\ \frac{J_z - J_x}{J_y} pr \\ \frac{J_x - J_y}{J_z} pq \end{bmatrix} + \begin{bmatrix} \frac{\tau_\phi}{J_x} \\ \frac{\tau_\theta}{J_y} \\ \frac{\tau_\psi}{J_z} \end{bmatrix} \tag{4}$$

Here, the control input for this model is $\mathbf{u} = \begin{bmatrix} F_c & \tau_\phi & \tau_\theta & \tau_\psi \end{bmatrix}^T$.

2.3. Trajectory Control using Linear Quadratic Regulator

For the purpose of estimation, a reference trajectory is required. The quadcopter is controlled to follow that trajectory using Linear Quadratic Regulator (LQR) controller. The trajectory controller and the attitude controller return the final control input to the quadcopter by minimizing the error between the commanded states (‘c’ in the subscript) and the estimated states (hat operator). Figure 1 shows the block diagram of the trajectory control.

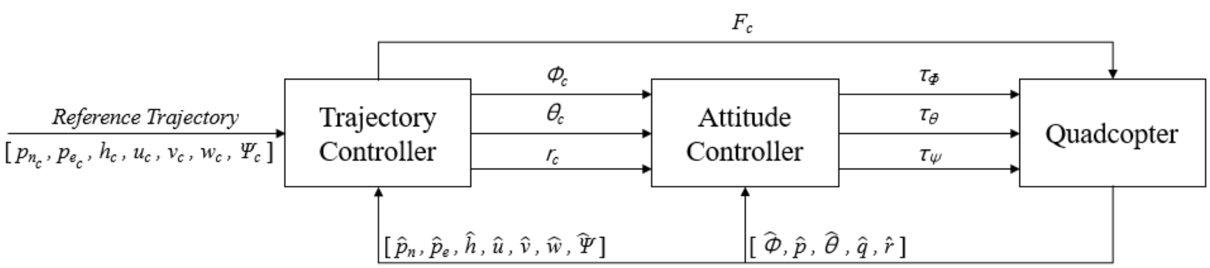

Figure 1. Flowchart of the Trajectory Control

2.4. EKF - SLAM

System dynamics in the presence of model noises and uncertainties are used to solve the SLAM problem, that is to determine the whole history of the quadcopter states and the landmark locations, given the Inertial Measurement Unit (IMU), LiDAR, and attitude measurements. SLAM consists of three basic operations, which are reiterated at each step [6].

I. The Quadcopter moves, reaching a new point of view of the map. Due to sensor and process noise, this motion increases the uncertainty of the quadcopter’s position. The motion model can be defined as:

$$\mathbf{R}_{i+1} = f(\mathbf{R}_i, \mathbf{u}, \mathbf{n}) \tag{5}$$

II. The Quadcopter discovers interesting features in the environment, called landmarks. Because of error in the exteroceptive sensor - LiDAR, the location of the landmarks will be uncertain. Additionally, the robot’s location is already uncertain. The quadcopter computes the coordinates of a newly discovered landmark, in the inertial frame, with the following mathematical model:

$$\mathbf{L}_k = g(\mathbf{R}, \mathbf{y}_k) \tag{6}$$

III. The Quadcopter observes landmarks than had been previously mapped and use them to correct both self- localization and the position of the observed landmarks, decreasing both localization and landmark uncertainties. The quadcopter obtains the measurement by the following mathematical model:

$$\mathbf{y}_k = h(\mathbf{R}, \mathbf{L}_k, \mathbf{v}) \tag{7}$$

With these three models combined with an estimation technique, an automated solution to SLAM can be achieved. The estimator is also responsible to reduce uncertainties in the IMU and LiDAR measurements. And since our dynamical model and measurement model is non-linear and sequential, EKF is proposed for localization and mapping. In EKF-SLAM, the total states vector, $\mathbf{X}=\begin{bmatrix}\mathbf{R} & \mathbf{M}\end{bmatrix}^T$, is maintained by EKF throughout the process of prediction, correction, and landmark initialization. An outline of the process is given in Fig. 2.

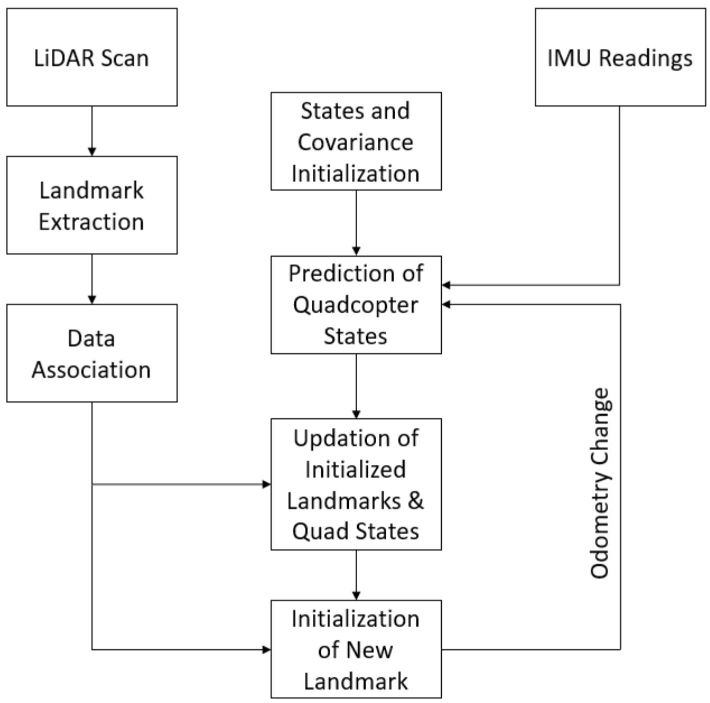

Figure 2. EKF-SLAM Flowchart

2.4.1. State and Covariance Initialization

The states and covariance are initialized as

$$\hat{\mathbf{X}} = \begin{bmatrix}\hat{\mathbf{R}} \\ \hat{\mathbf{M}}\end{bmatrix} = \begin{bmatrix}\hat{\mathbf{R}} \\ \hat{\mathbf{L}}_1 \\ \vdots \\ \hat{\mathbf{L}}_n\end{bmatrix}, \qquad \mathrm{P} = \begin{bmatrix}\mathrm{P}_{RR} & \mathrm{P}_{RM} \\ \mathrm{P}_{MR} & \mathrm{P}_{MM}\end{bmatrix} = \begin{bmatrix}\mathrm{P}_{RR} & \mathrm{P}_{RL_1} & .. & \mathrm{P}_{RL_n} \\ \mathrm{P}_{L_1R} & \mathrm{P}_{L_1L_1} & .. & \mathrm{P}_{L_1L_n} \\ \vdots & \vdots & . & \vdots \\ \mathrm{P}_{L_nR} & \mathrm{P}_{L_nL_1} & .. & \mathrm{P}_{L_nL_n}\end{bmatrix} \tag{8}$$

Here, $\hat{\mathbf{X}}$ starts with no landmarks, therefore $\hat{\mathbf{X}} = \hat{\mathbf{R}}$. Also, the initial quadcopter location is assumed to be the true location of the quadcopter. Therefore, we also take the initial covariance matrix, P, as close to zero (absolute certainty).

2.4.2. Prediction of Quadcopter States

Only quadcopter states will be predicted as our landmarks are stationary. The EKF prediction step can be written as:

$$\dot{\hat{\mathbf{X}}}(t) = f(\hat{\mathbf{X}}(t), \mathbf{u}(t), t) \tag{9}$$

$$\dot{\mathrm{P}}(t) = \mathrm{F}(t)\mathrm{P}(t) + \mathrm{P}(t)\mathrm{F}^T(t) + \mathrm{G}(t)\mathrm{Q}(t)\mathrm{G}^T(t), \qquad \mathrm{F}(t) = \frac{df(t)}{d\mathbf{X}} \tag{10}$$

Here, $\mathrm{G}(t)$ is a column matrix of ones, and $\mathrm{Q}(t)$ is the process noise covariance matrix.

2.4.3. Updation of Initialized Landmarks & Quadcopter States

LiDAR measurements are used to update the states of the quadcopter as well as the states of the initialized landmarks which are being sensed at that moment. Landmarks observations are processed in EKF one by one. The gain update step can be written as:

$$\mathrm{K}_k = \mathrm{P}_k^- \mathrm{H}_k^T(\hat{\mathbf{X}}_k^-) \left[\mathrm{H}_k(\hat{\mathbf{X}}_k^-) \mathrm{P}_k^- \mathrm{H}_k^T(\hat{\mathbf{X}}_k^-) + \mathrm{R}_k \right]^{-1}, \qquad \mathrm{H}_k^T(\hat{\mathbf{X}}_k^-) = \frac{dh_k}{d\mathbf{X}} \tag{11}$$

K_k is the Kalman gain and R_k is the measurement error covariance matrix. The correction step based on the measurement model can be written as:

$$\hat{\mathbf{X}}_k^+ = \hat{\mathbf{X}}_k^- + \mathrm{K}_k \left[\mathbf{y}_k - h(\hat{\mathbf{X}}_k^-) \right] \tag{12}$$

$$\mathrm{P}_k^+ = \left[I - \mathrm{K}_k \mathrm{H}_k(\hat{\mathbf{X}}_k^-) \right] \mathrm{P}_k^- \tag{13}$$

2.4.4. Initialization of the New Landmarks

Landmark initialization is done when the quadcopter discovers new landmarks that are yet to be incorporated into the map. Therefore, this operation results in an increase of the state vector's size. The landmark covariance matrix P_{LL} and its covariance with the rest of the map P_{Lx} is updated as follows:

$$\mathrm{P}_{Lx} = \mathrm{G}_R \left[\mathrm{P}_{RR} \mathrm{P}_{RM} \right], \qquad \mathrm{G}_R = \frac{\partial g(\mathbf{R}, \mathbf{y}_k)}{\partial \mathbf{R}} \tag{14}$$

$$\mathrm{P}_{LL} = \mathrm{G}_R \mathrm{P}_{RR} \mathrm{G}_R^T + \mathrm{G}_{y_{k+1}} \mathrm{R} \mathrm{G}_{y_{k+1}}^T, \qquad \mathrm{G}_{y_k} = \frac{\partial g(\mathbf{R}, \mathbf{y}_k)}{\partial \mathbf{y}_k} \tag{15}$$

3. Simulation Results

3.1. Environment Setup

For the simulation, the considered quadcopter's mass and inertia properties are tabulated in the Tab. 1. A reference trajectory of a spiral moving up along a cylinder is generated with a radius of 5 m and the pitch of 0.1 m. The simulation time is set to 50 s and the sampling rate for all the sensors is

Table 1. Quadcopter Properties

Parameter	Value
m	1.56 kg
J_x	0.114700 kgm^2
J_y	0.057600 kgm^2
J_z	0.171200 kgm^2

assumed as 10 Hz. A total of 50 landmarks are generated around the trajectory of the quadcopter with the azimuth range as $\begin{bmatrix}-180 & 180\end{bmatrix}$ deg, elevation range as $\begin{bmatrix}-50 & 50\end{bmatrix}$ deg, and radial range as $\begin{bmatrix}8 & 20\end{bmatrix}$ m in the inertial frame. Similarly, the range of the LiDAR sensor is selected as 20 m with the azimuth range as $\begin{bmatrix}-45 & 45\end{bmatrix}$ deg, and elevation range as $\begin{bmatrix}-30 & 30\end{bmatrix}$ deg in the body frame.

For each of the sensors and estimator, the noise assumed is tabulated in the Tab. 2.

Table 2. Sensor Measurements Error

Sensor	Measurement Noise
LiDAR (Range)	0.1 m
LiDAR (Azimuth & Elevation)	0.01 rad
Accelerometer	0.1 m/s^2
Gyroscope	0.01 rad/s
Attitude Estimation (Low Level Filter)	0.01 rad

3.2. Evaluation of the Algorithm

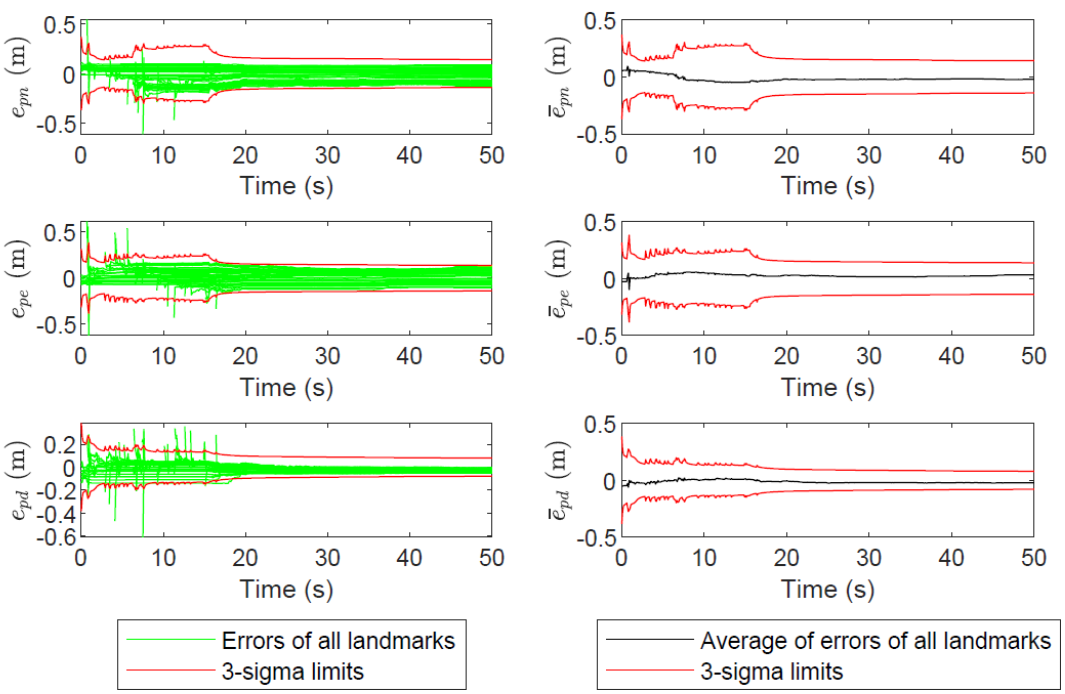

Figure 3. Estimation error of landmarks (Left) and Average of estimation error (Right)

Figure 3 shows the estimation error of all the landmarks bounded by the average 3-sigma boundary on the left and the average estimation error on the right.

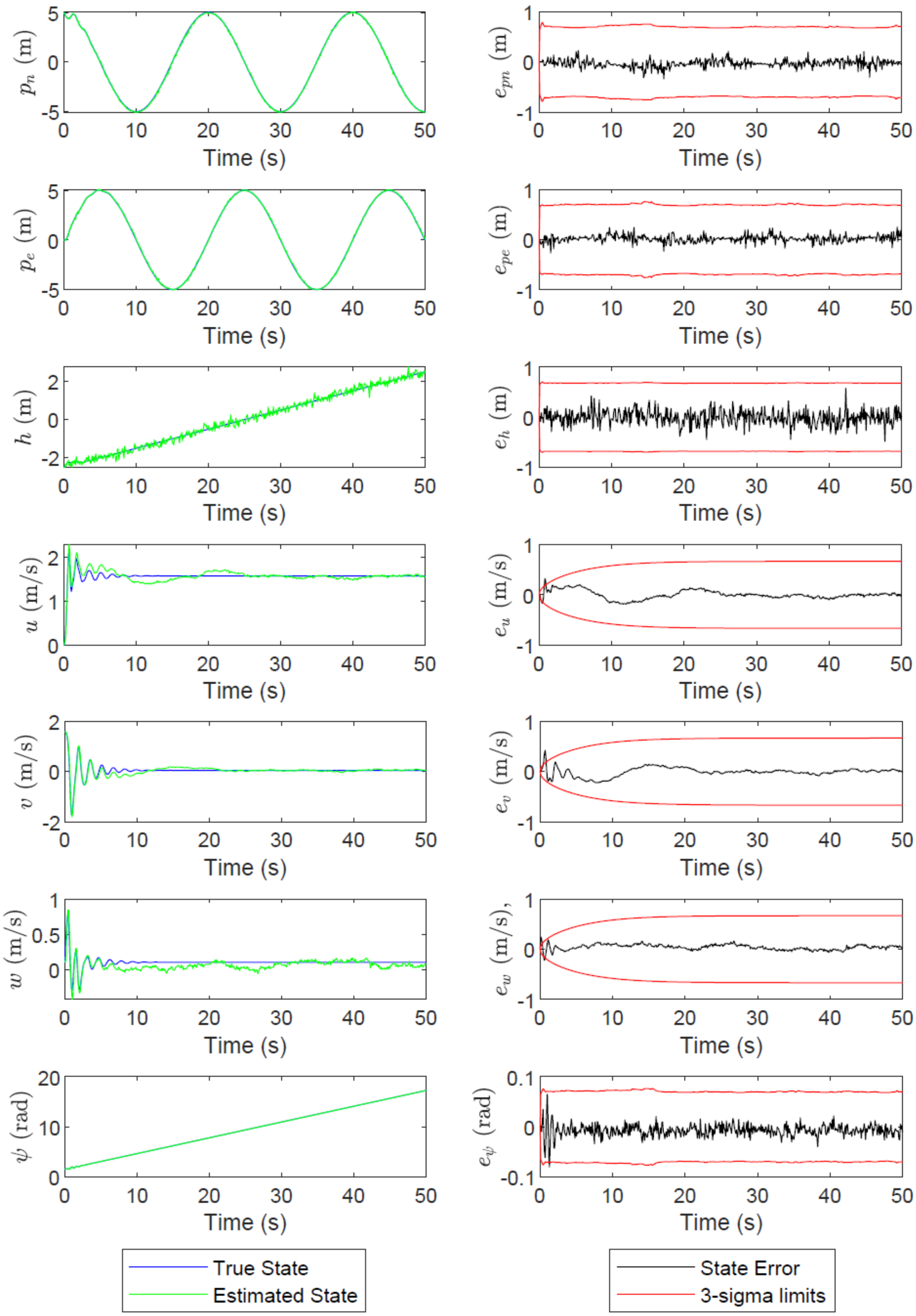

Figure 4. Estimated vs True states (Left) and Estimation error of quadcopter states (Right)

Figure 4 (Left) shows that with the proper choice of initial $\hat{P}$ and Q, the trajectory estimation is close to the actual path derived from the LQR control but with some fluctuating errors. Figure 4 (Right) verifies that estimation errors are bounded in the 3-sigma boundary. Thus, the EKF-SLAM proposed here works well for the estimation of both the landmarks and the quadcopter. The root mean square errors for the positions of the quadcopter and landmarks are calculated to be 10.50 % and 3.33 %, respectively.

4. Conclusion

In this paper, a dynamic Extended Kalman Filter (EKF) algorithm is proposed for the Light Detection and Ranging (LiDAR) and inertial measurement unit-equipped quadcopter system for the three-dimensional Simultaneous Localization and Mapping (SLAM) problem. In addition to the conventional EKF, initialization of landmark states is done in each iteration by the inverse observation model of the LiDAR sensor. A field of view of LiDAR and sensor noises are added to the simulation to imitate the real-world system. Then the results of the quadcopter states estimation in SLAM are discussed. Estimation errors of landmarks are also plotted with respect to simulation time. For this work, we assumed the landmark coordinates in the inertial frame are readily available with the data processing on LiDAR sensor readings. In future works, we can create a robust observation model for the landmark coordinates. Bias and drift can be added to the observation model and estimated in the EKF along with other states. Hardware experimentation can also be performed to know if the proposed algorithm works as per the simulation.

5. References

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